SPHERICALLY SYMMETRIC COSMIC STRINGS in a SCALAR-TENSOR THEORY

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Abstract: Spherically Symmetric cosmological models with cosmic string source are obtained in a scalar- tensor theory of gravitation proposed by Saez and Ballester [1]. The models obtained give us spherically symmetric geometric (Nambu) string, ρ - string and Reddy string[2] in Saez-Ballester theory. Some physical properties of the models are also discussed.

Keywords: Spherically symmetric, cosmic string, scalar tensor theory.

1. INTRODUCTION

The fact that general relativity is not in accordance with Mach's principle has stimulated numerous attempts to develop scalar tensor theories of gravity which incorporate Mach's principle. Brans and Dicke [3] introduced a scalar-tensor theory of gravitation involving a scalar function in addition to the familiar general relativistic metric tensor. In this theory the scalar field has the dimension of inverse of the gravitational constant and its role is confined to its effects on gravitational field equations. Subsequently Saez-Ballester [1] developed a scalar- tensor theory in which the is couple with a dimensionless scalar field. Inspite of the dimensionless character of the scalar field an antigravity regime appears. This theory suggests a possible way to solve the missing matter problem in non-flat FRW cosmologies.

Einstein's general theory of relativity has been successful in describing gravitational phenomena. It has also served as a basic for models of the universe. However, since Einstein first published his theory of gravitation, there have been many criticisms of general relativity because of the lack of certain' desirable' features in the theory.. For example Einstein himself pointed out that general relativity does not account satisfactorily for inertial properties of matter i. e. Mach's principle is not substantiated by general relativity. So in recent years there has been lot of interest in several alternative theories of gravitation. The most important among them are scalar-tensor theories of gravitation formulated by Brans-Dick [3], Nordvedt [4] and Saez-Ballester [1]. All versions of the scalar- tensor theories are based on the introduction of a scalar field ϕ into the formulation of general relativity, this scalar field together with the metric tensor field then forms a scalar tensor field representing the gravitational field.

It is well known that a gravitational scalar field, beside then metric of the space-time must exist in the frame work of the present unified theories. Hence there has been must interest in scalar- theories of gravitation. These theories have importance in the early universe where it is expected that the coupling to the matter of scalar field would be of the same order as that of metric, although the scalar coupling is negligible in the present time.

In Brans- Dicke theory the scalar field ϕ has the dimension of the inverse of a gravitational constant which interacts equally with all forms of matter (with the exceptional of electromagnetism) while in Saez –Ballester theory the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of weak fields and suggests a possible way to solve missing matter problem in non-falt FRW cosmologies, Seaz[1], Singh and Agrawal [5], Sri Ram and Tiwan[6] Venkateswara Rao[7] are some of the authors who have been investigated several aspects of the Saez-Ballester [1]scalar –tensor theory. The latest inflationary models (Mathiazhagan and Johri [8], possible "graceful exist" problem Pimental [9] and extended chaotic inflation Linde [10] are based on Brans-Dicke and other scaler – tensor theories.

Cosmic string are one dimensional topological defects associated with spontaneous symmetry breaking whose plausible production site is cosmological phase transitions in the early universe. Kibble and Vilenkin [11,12], Letelier [13], Velenkin [14], Chakraborty and chakraborty [15 12], Bali and Singh [16] and Krori et al.[17, 18] are some of the authors who have investigated string cosmological models in general relativity while Mahanta and Mukherjee [19] , Bhattacharjee and Baruah [20], Rahaman et al. [21,22] ,Barros et al. [23],Reddy [24,25,26] and Sen [27] are some of the authours who have studied string cosmological models in alternative theories of gravitation. In particular Reddy [2] has discussed an exact Bianchi type-I string cosmological model (Reddy string) in Saez- Ballester scalar – tensor theory. Also, very recently,Reddy and Rao [27] obtained axially symmetric cosmological models in Lyra[28] manifold with string source.

In this paper, we consider a Spherically symmetric cosmological model in Saez- Ballester scalar- tensor theory of gravitation in the presence of cosmic string source . We obtain cosmological models which represent Nambu string, p-string by Letelier [29] and Reddy string [23,2].

2. METRIC FIELD EQUATIONS

We consider the spherically symmetric metric

$$ds^2 = -dt^2 + a^2 dr^2 + b^2 d\Omega^2 \tag{1}$$

Where a and b are function of t only

The field equations given by Saez- Ballester [1] for the combined scalar and tensor fields are

$$G_{ij} - \omega\phi^n(\phi,_i \phi,_j - \frac{1}{2} g_{ij}\phi,_k \phi^{,k}) = -8\pi T_{ij} \tag{2}$$

and the scalar field ϕ satisfies the equation

$$2\phi^n \phi^{;i}_{,i} + n\phi^{n-2}\phi_{,k}\phi^{,k} = 0 \tag{3}$$

Where

n is an arbitrary exponent,

ω is dimensionless constant and

$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R$ is the usual Einstein tensor.

In the case of cosmic string dust source, the energy momentum tensor T_{ij} is given by

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \tag{4}$$

here

ρ is the rest energy density of cloud of strings with particles attached to them,

λ the tension density of strings,

u^i the cloud four velocity and x^i is the direction of anisotropy.

Orthonomalisation of u^i and x^i given as

$$u^i u_i = -1 = -x^i x_i \quad , \quad u^i x_i = 0 \tag{5}$$

Also, we have

$$\rho = \rho_p + \lambda,$$

where ρ_p is the rest energy density of particles and

x^i to be along x- axis, so that

$$x^i = (0, a^{-1}, 0, 0)$$

The simplified form of the Bianchi identity for the above metric is

$$\rho_4 + \frac{a_4}{a}(\rho - \lambda) + 2\rho\frac{b_4}{b} = 0 \tag{6}$$

In the co-moving coordinate system, we have from equation (4) and (5)

$$T_4^4 = -\rho \ , T_1^1 = -\lambda \qquad T_2^2 = 0 = T_3^3$$

$$T_i^j = 0 \qquad for \quad i \neq j \tag{7}$$

$$\frac{2b_{44}}{b} + \frac{1}{b^2} + \frac{{b_4}^2}{b^2} - \omega\phi^n \frac{{\phi_4}^2}{2} = 8\pi\lambda \tag{8}$$

$$\frac{a_{44}}{a} + \frac{b_{44}}{b} + \frac{a_4 b_4}{ab} - \omega\phi^n \frac{{\phi_4}^2}{2} = 0 \tag{9}$$

$$\frac{2a_4 b_4}{ab} + \frac{1}{b^2} + \frac{{b_4}^2}{b^2} + \omega\phi^n \frac{{\phi_4}^2}{2} = 8\pi\rho \tag{10}$$

$$\frac{\phi_{44}}{\phi} + \frac{a_4}{a} + 2\frac{b_4}{b} + \frac{n}{2}\frac{\phi_4}{\phi} = 0 \tag{11}$$

The physical quantities that are of importance in cosmology are Proper Volume V, expansion scalar θ and shear scalar σ^2 and have the following expressions for the metric

$$V = ab^2 \sin\theta$$

$$\theta = \frac{a_4}{a} + 2\frac{b_4}{b} \tag{11}$$

$$\sigma^2 = \frac{2}{3}\left(\frac{b_4}{b} - \frac{a_4}{a}\right)^2 \tag{12}$$

Here the suffix 4 after field variable represents ordinary differentiation with respect to time

3. COSMIC STRING MODELS

Here we have four independent field equation (7) to (10) connecting five unknown quantities $a, b, \rho, \lambda, \phi$.

Therefore, in order to obtain exact solutions. We must need one more relation connecting the unknown quantities. We assume the relation as the equation of state. Since the field equations are highly non- linear, we also assume an analytic relation between metric coefficients (scale factors) as,

$$a = \mu b^n \tag{13}$$

to get determinate solution by Chakraborty et al.[15]

case i): Geometric string

Here the equation of state of Nambus (Letelier [13]) is

$$\rho = \lambda \tag{14}$$

Then the field equations admits an exact solution

$$a(t) = \mu[(n+2)(k_1 t + k_2)]^{\frac{n}{n+2}}$$

$$b(t) = [(n+2)(k_1 t + k_2)]^{\frac{1}{n+2}}$$

$$\rho = \lambda = \frac{1}{(n+2)^{\frac{2}{n+2}}}(k_1 t + k_2)^{\frac{-2}{n+2}} \tag{15}$$

$$\phi^{\frac{n+2}{2}} = \frac{-k_3}{2(n+1)}(n+2)^{\frac{1}{n+2}}(k_1 t + k_2)^{\frac{-(n+1)}{(n+2)}} + \frac{k_4(n+2)}{2} \tag{16}$$

where k_i's are integrating constants. The corresponding string model of the solution can be written, through a proper choice of constant of integration and coordinates, as

$$ds^2 = -dt^2 + \mu^2[(n+2)T]^{\frac{2n}{n+2}} dr^2 + [(n+2)T]^{\frac{2}{n+2}} d\Omega^2 \tag{17}$$

Thus equation (17) represents the geometric (Nambus) string cosmological model in the framework of scalar – tensor theory formulated by Saez-Ballester [1]

4. SOME PHYSICAL PROPERTIES OF THE MODEL

The physical quantities that are important in cosmology are proper volume V, expansion scalar θ, shear scalar σ^2 and have the following expressions for the string model(17)

$$V = (-g)^{1/2} = \mu(n+2)T\sin\theta \tag{18}$$

$$\theta = \frac{k_1}{T} \quad ; \quad \sigma^2 = \frac{2(n-1)^2 k_1^2}{3(n+2)^2 T^2} \tag{19}$$

$$q = 2 \tag{20}$$

The string density and the tension density in the model (14) are

$$\rho = \lambda = \frac{1}{(n+2)^{\frac{2}{n+2}}} T^{\frac{-2}{n+2}} \tag{21}$$

The Saez – Ballester scalar field in the model is

$$\phi^{\frac{n+2}{2}} = \frac{-k_3}{2(n+1)}(n+2)^{\frac{1}{n+2}} T^{\frac{-(n+1)}{(n+2)}} + \frac{k_4(n+2)}{2} \tag{22}$$

The cosmic string model (17) has no initial singularity, while the energy density, tension density of the string and the scalar field diverge at $T = 0$. As $T \to \infty$ the spatial volume tends to infinity while θ, σ, ρ *and* λ tends to zero. The positive value of the deceleration parameter shows that the model deceleration in standard way. Also, since $\lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0$, the model does not approach isotropy for large value of T.

Case ii): p-string

In this case the equation of state of Takabayasi (30) is

$$\rho = (1+\omega)\lambda \tag{23}$$

With $\omega > 0$, a constant, In this case, again, assuming the relation between metric coefficient, given by the equation (13),

It is interesting to observe that we obtain the same string model given by equation (17) with the same physical and kinematical parameters and properties except that

$$\rho = (1+\omega)\lambda = [(n+2)T]^{\frac{-2}{n+2}} \tag{24}$$

Case iii): Reddy string

Recently, Reddy [2, 23] has obtained string cosmological models in Brans- Dicke [3] and Saez- Ballester [1] scalar-tensor theories of gravitation with equation of state for string density

$$\rho + \lambda = 0 \tag{25}$$

ie. Sum of the rest energy density and tension density for a cloud of string vanishes. The relation (25), in a way, is analogous to $\rho + p = 0$ equation of state in general relativity which represent the false vacuum.

In this case, again assuming the relation between scale factors given by (13), we get the same string model given by (17), with the same physical and kinematical parameter except that

$$\rho = -\lambda = [(n+2)T]^{\frac{-2}{n+2}} \tag{26}$$

Comparing the cosmic string (17) with the string cosmological model obtain by Reddy [2, 23] in Brans- Dicke theory, we observe that the model in the theory inflates. Also, the physical quantities like energy density and tension density of the string diverge in this theory while they do not in Brans- Dicke theory.

5. C0NCLUSION

We have studied cosmological models generated by a cloud of strings with particles attached to them in an spherically symmetric space – time in the frame work of Saez-Ballester [1] scalar- tensor theory of gravitation. The models obtained represent Nambu string, p- string and Reddy string in this theory. It is interesting to note that the models are identical in the above three cases. The models are free from initial singularity. They are expanding, anisotropic, shearing, non- rotating in the standard way. Also, we find that in these models, the rest energy density, tension density of the string and the scalar field have singularities at the initial moment of creation of the universe and hence we do not have any knowledge of the state of cosmic strings at that instant. Also, a comparison of the models obtained, here, is model with the string models of Brans- Dicke theory and general relativistic theory.

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